Laboratorium sterowania adaptacyjnego

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C2 MODEL-IDENTIFICATION ADAPTIVE CONTROL – MIAC (UKŁAD STEROWANIA ADAPTACYJNEGO TYPU MIAC)

This exercise is devoted to the adaptive control design problem in the MIAC (Model-Identification Adaptive Control) scheme for the exemplary plant and to verification of the designed control system in the Matlab-Simulink environment. We will apply a deterministic approach to the controller synthesis, which is reasonable under assumption that the stochastic noise disturbing the plant is negligibly small (signal-to-noise ratio is sufficiently high).

1 Description of the plant

Let us consider the *aero-plant* in a form of the aircraft roll dynamics as illustrated in Fig. 1. Due to the differential deflection δ_a (expressed in [rad]) of ailerons on the left and right sides of

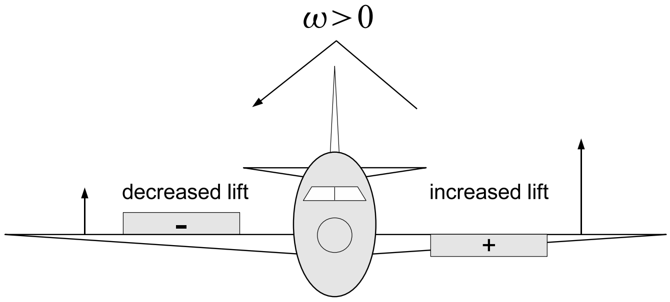

Figure 1: Front view of the aircraft illustrating the roll motion caused by ailerons deflection (based on E. Lavretsky, K. A. Wise: Robust and Adaptive Control with Aerospace Applications, Springer, London, 2013)

the aircraft body, it is possible to change the aircraft roll-rate ω expressed in [rad/s]. Locally (in a small vicinity of zero steady-state conditions), we can approximate the roll-rate dynamics by the following linear differential equation:

$$\dot{\omega} = -a_o\omega + b_o\delta_a, \tag{1}$$

where a_o and b_o are the true (unknown in practice and possibly time-varying) parameters of the plant. We can rewrite equation (1) in the following form

$$T_o\dot{y} + y = k_o u, \qquad \text{where} \qquad y \triangleq \omega, \quad u \triangleq \delta_a, \quad T_o = \frac{1}{a_o}, \quad k_o = \frac{b_o}{a_o} \tag{2}$$

where we assume that only input u and output y are available for measurements, while T_o and k_o denote, respectively, the true time-constant and the true dc-gain of the plant (unknown in practice and possibly time-varying). Note that values of the control input $u = \delta_a$ are inherently constrained to the range $[-\pi; \pi]$ rad due to physical interpretation of δ_a.

2 Control performance requirements

We are interested in designing the MIAC system for the aero-plant represented by equation (2) with unknown parameters $T_{\mathrm{o}}, k_{\mathrm{o}}$ which guarantees satisfaction of the following prescribed performance requirements:

R1. signal $y_r(t) = \omega_r(t)$ is a bounded time-varying reference trajectory for the aircraft roll rate such that $\dot{y}_r(t)$ exists and is bounded,

R2. tracking error $e(t) \triangleq y_r(t) - y(t)$ asymptotically converges to zero, that is $\lim_{t\to\infty} e(t) = 0$, with no overshoot during the transient stage,

R3. settling time $T_{s1\%}$ of the closed-loop system satisfies $T_{s1\%} = \alpha$ for $\alpha > 0$ expressed in [s].

3 Control system design

3.1 Step 1: design of the identification block

Let us introduce the continuous-time domain model of the plant in the form

$$T\dot{y} + y = ku, \tag{3}$$

where T and k are the model parameters which need to be estimated (compare (2)). To keep the continuous-time domain model structure during parametric identification process we shall apply the SVF (State Variable Filters) method for preparation of the linear-regression-form model useful in practice, that is

$$y_F(nT_a) = \boldsymbol{\varphi}^\top(nT_a)\boldsymbol{p}, \qquad \boldsymbol{\varphi}^\top(nT_a) = [-\dot{y}_F(nT_a)\ u_F(nT_a)], \quad \boldsymbol{p} = \begin{bmatrix} T \\ k \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}, \tag{4}$$

where $y_F(nT_a)$, $\dot{y}_F(nT_a)$ and $u_F(nT_a)$ denote the appropriately filtered signals sampled next with sampling period T_a, while $\boldsymbol{p}$ is a vector of parameters which shall be estimated using the RLS (Recursive Least Squares) method.

3.2 Step 2: design of the controller block

We assume the following structure of a controller expressed in the operator domain

$$U(s) \triangleq U_R(s) + U_{FF}(s) = G_R(s)E(s) + G_{FF}(s)Y_r(s) \triangleq w_1 E(s) + (w_2 + w_3 s)Y_r(s), \tag{5}$$

which can be rewritten in the time domain by using interpretation of the s operator:

$$u(t) \triangleq w_1 e(t) + w_2 y_r(t) + w_3 \dot{y}_r(t) = [e(t)\ y_r(t)\ \dot{y}_r(t)]\boldsymbol{w}, \tag{6}$$

where $e(t) = y_r(t) - y(t)$ is the tracking error, while

$$\boldsymbol{w} = [w_1\ w_2\ w_3]^\top \tag{7}$$

is a vector of controller parameters. It is worth noting that (5) is a combination of the proportional regulator represented by G_R with the feedforward controller represented by G_{FF} which utilizes the reference trajectory and its time derivative (the latter is bounded and available in practice according to requirement R1). In the next subsection it will be shown that the controller structure proposed in (6) allows satisfying performance requirements R2 to R3.

3.3 Step 3: design of the synthesis block

Now, the objective is to derive the synthesis equations $\boldsymbol{w} = \boldsymbol{w}(\boldsymbol{p}_\mathrm{o})$, that is, we need to find equations $\boldsymbol{w} = \boldsymbol{w}(T_\mathrm{o}, k_\mathrm{o})$ which relate controller parameters (7) with parameters of the plant (2) in order to satisfy performance requirements R2 to R3. To this aim, let us derive and analyze dynamics of the tracking error $e(t)$ valid in the closed-loop system with controller (6) and the plant represented by (2). Combining (2) with (6), and utilizing the fact that $e(t) = y_r(t) - y(t)$, allows us to write (we omit the time argument for simplicity of notation):

$$\begin{aligned} T_\mathrm{o}\dot{y} + y &= k_\mathrm{o}(w_1 e + w_2 y_r + w_3 \dot{y}_r), \\ \frac{T_\mathrm{o}}{k_\mathrm{o}}(\dot{y}_r - \dot{e}) + \frac{1}{k_\mathrm{o}}(y_r - e) &= w_1 e + w_2 y_r + w_3 \dot{y}_r, \\ \frac{T_\mathrm{o}}{k_\mathrm{o}}\dot{e} + \frac{1 + k_\mathrm{o}w_1}{k_\mathrm{o}} e &= \left(\frac{T_\mathrm{o}}{k_\mathrm{o}} - w_3\right)\dot{y}_r + \left(\frac{1}{k_\mathrm{o}} - w_2\right) y_r. \end{aligned} \tag{8}$$

Requirement R2 can be satisfied by making the right-hand side of (8) equal to zero by selecting

$$w_2 \triangleq \frac{1}{k_\mathrm{o}}, \qquad w_3 \triangleq \frac{T_\mathrm{o}}{k_\mathrm{o}}. \tag{9}$$

Now, equation (8) reduces to

$$\tau\dot{e} + e = 0, \quad \text{where} \quad \tau = \frac{T_\mathrm{o}}{1 + k_\mathrm{o}w_1}. \tag{10}$$

If τ is positive, then solution of equation (10) exponentially converges to zero for any initial condition $e(0)$ without an overshoot, that is,

$$e(t) = e(0)\exp(-t/\tau)$$

with time constant τ (requirement R2 is satisfied). The settling time $T_{s1\%}$ can be expressed as a five times the time constant τ, thus requirement R3 can be expressed as follows:

$$T_{s1\%} = 5\tau = \alpha \qquad \overset{(10)}{\Longrightarrow} \qquad \frac{5T_\mathrm{o}}{1 + k_\mathrm{o}w_1} = \alpha \qquad \Longrightarrow \qquad w_1 = \frac{5T_\mathrm{o} - \alpha}{k_\mathrm{o}\alpha}. \tag{11}$$

Hence, by computing parameter w_1 according to (11), one can ensure satisfaction of requirement R3.

Collecting together design equations (9) and (11) yields the synthesis equations for controller (6) which guarantees satisfaction of requirements R2 and R3:

$$\boldsymbol{w} = \boldsymbol{w}(T_\mathrm{o}, k_\mathrm{o}) = \begin{bmatrix} w_1(T_\mathrm{o}, k_\mathrm{o}) & w_2(T_\mathrm{o}, k_\mathrm{o}) & w_3(T_\mathrm{o}, k_\mathrm{o}) \end{bmatrix}^\top = \begin{bmatrix} \frac{5T_\mathrm{o} - \alpha}{k_\mathrm{o}\alpha} & \frac{1}{k_\mathrm{o}} & \frac{T_\mathrm{o}}{k_\mathrm{o}} \end{bmatrix}^\top. \tag{12}$$

3.4 Step 4: application of the CE principle in the resultant control law

The vector of parameters determined by synthesis equations (12) cannot be used in practice because the true plant parameters T_o and k_o are unknown. Thus, we apply the Certainty Equivalence (CE) principle replacing (12) with a more practical version taking

$$\boldsymbol{w} = \boldsymbol{w}(\hat{T}, \hat{k}) = \begin{bmatrix} w_1(\hat{T}, \hat{k}) & w_2(\hat{T}, \hat{k}) & w_3(\hat{T}, \hat{k}) \end{bmatrix}^\top = \begin{bmatrix} \frac{5\hat{T} - \alpha}{\hat{k}\alpha} & \frac{1}{\hat{k}} & \frac{\hat{T}}{\hat{k}} \end{bmatrix}^\top, \tag{13}$$

where $\hat{T}$ and $\hat{k}$ are the estimated plant parameters being the components of estimate $\hat{\boldsymbol{p}}$ computed according to the RLS method (see (4)). Substituting (13) into definition (6) gives the resultant MIAC-type adaptive control law expressed in the continuous time domain:

$$u(t) = \underbrace{\left(\frac{5\hat{T}(nT_a) - \alpha}{\alpha\hat{k}(nT_a)}\right)}_{w_1(\hat{\boldsymbol{p}}(nT_a))} e(t) + \underbrace{\left(\frac{1}{\hat{k}(nT_a)}\right)}_{w_2(\hat{\boldsymbol{p}}(nT_a))} y_r(t) + \underbrace{\left(\frac{\hat{T}(nT_a)}{\hat{k}(nT_a)}\right)}_{w_3(\hat{\boldsymbol{p}}(nT_a))} \dot{y}_r(t), \tag{14}$$

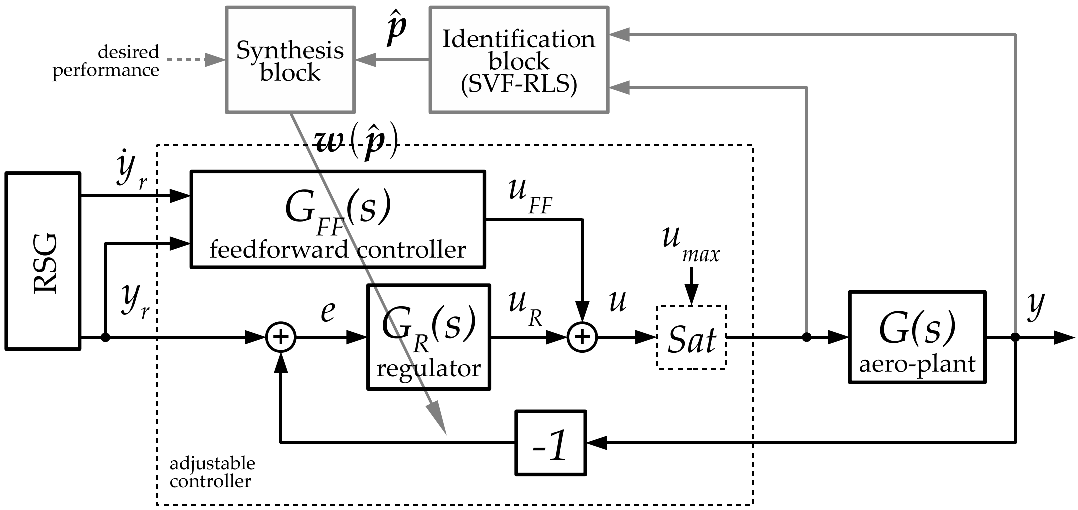

Figure 2: Block scheme of the MIAC system for the aero-plant; the blocks and arrows highlighted in grey constitute the adaptive loop in the control system while the black ones correspond to the conventional part of the control system (RSG = Reference Signals Generator)

where T_a denotes the sampling interval for the adaptive control loop. The resultant MIAC control system corresponding to control law (14) is presented in Fig. 2.

In the discrete-time domain (convenient for computer implementations) the control law (14) shall be rewritten as follows:

$$u(nT_c) = \underbrace{\left(\frac{5\hat{T}(nT_a) - \alpha}{\alpha\hat{k}(nT_a)}\right)}_{w_1(\hat{\boldsymbol{p}}(nT_a))} e(nT_c) + \underbrace{\left(\frac{1}{\hat{k}(nT_a)}\right)}_{w_2(\hat{\boldsymbol{p}}(nT_a))} y_r(nT_c) + \underbrace{\left(\frac{\hat{T}(nT_a)}{\hat{k}(nT_a)}\right)}_{w_3(\hat{\boldsymbol{p}}(nT_a))} \dot{y}_r(nT_c), \tag{15}$$

where T_c denotes the sampling interval for the conventional control loop. Note that estimate $\hat{\boldsymbol{p}}$ in equation (15) is updated with sampling time T_a and the control signal is computed with sampling time T_c; a suggested relation between the sampling times is

$$T_a \geqslant T_c. \tag{16}$$

Furthermore, the form of (15) clearly indicates that there is a possible danger of the closed-loop system instability when estimate $\hat{k}(nT_a)$ goes through zero (or is close to zero) during transients of an adaptation process. Therefore, in practical applications of control law (15) a designer should additionally implement appropriate supervision/safety nets which prevents this danger situation.

Finally, it is worth emphasizing that asymptotic convergence of tracking error to zero (requirement R2) can be expected for the controller designed and synthesized in points 3.2 and 3.3 when applying its continuous-time version defined by (14). Discrete-time controller version (15) allows only approximating requirement R2 for sufficiently small sampling interval T_c.

3.1 Parametric identification of the plant using the SVF-RLS method.

- Open the file `AeroPlantMIAC.mdl` which contains the plant (2) and the reference signal generator (RSG). The RSG block produces two types of the reference trajectory $y_r(t)$ and its time derivative $\dot{y}_r(t)$:

$$\text{TYPE 1:} \quad y_r(t) \triangleq Y_r \sin(\omega_r t), \qquad \dot{y}_r(t) = Y_r \omega_r \cos(\omega_r t), \tag{17}$$

$$\text{TYPE 2:} \quad y_r(t) \triangleq Y_r \mathrm{rect}(\omega_r t), \qquad \dot{y}_r(t) = 0, \tag{18}$$

  where $\mathrm{rect}(\omega_r t)$ represents a symmetric rectangular signal with the unit amplitude and frequency ω_r rad/s.

- Initialize the following global variables: `Ta`=0.05 s, `Tc`=0.001 s, `sigma2e`=0.0, which represent, respectively, the sampling interval of the adaptation loop, the sampling interval of the conventional control loop, and the variance of a stochastic noise disturbing the plant.

- On the scheme in file `AeroPlantMIAC.mdl` implement the RLS identification block for the plant dynamics applying the SVF method to filter the appropriate signals. Use the `Zero-Order Hold` blocks to sample all the signals needed for identification purposes. Choose a value of time-constant T_F used in the SVF filters to obtain an acceptable quality of identification – start using $T_F = 1.5T_a$. Implement the synthesis block using equation (13). Ensure that all the blocks of the adaptive loop are synchronized with the same sampling time $T_a \geqslant T_c$.

- Check the identification quality exciting the plant in the **open-loop** (without a controller) by applying input signals taken from the RSG block with the following parameters: $Y_r = 1.0$ rad/s, $\omega_r = 0.5$ rad/s. Analyze time plots of estimates $\hat{\boldsymbol{p}}$ and values of controller parameters $\boldsymbol{w}(\hat{\boldsymbol{p}})$. Repeat the identification process under noisy conditions initializing variable `sigma2e` from the set

$$\texttt{sigma2e} \in \{0.01; 0.1; 1.0\}.$$

3.2 Closed-loop control of the plant without adaptation.

- Implement the discretized controller (15) in the Simulink environment using **fixed values** of parameters $\boldsymbol{w}$ =const like in the conventional control system – take the following exemplary values:

$$\boldsymbol{w} = [w_1 \; w_2 \; w_3]^\top = [5.0 \quad 1.0 \quad 1.0]^\top. \tag{19}$$

  Ensure that all blocks of the conventional control system (that is, RSG and the controller) are synchronized with the same sampling time T_c (note, however, that the plant is still a continuous-time process!).

- Run the conventional control system with fixed parameters (19) and analyze the resultant control quality for both types of a reference trajectory generated by the RSG block – see (17)-(18) – using parameters: $Y_r = 0.15$ rad/s, $\omega_r = 0.25$ rad/s.
  Important: for the analysis purposes check the time plots of the tracking error e as well as the control input u, and compare output y with trajectory y_r on the same plot.
  Does the system satisfy performance requirements R2 and R3? Is it easy to manually re-tune the controller to meet requirements R2 and R3?

3.3 Adaptive control of the plant in the MIAC scheme.

- Modify the controller to enable on-line retuning of its parameters $\boldsymbol{w}$ according to the adjustment rule (13) (compare (15)) and prescribing $\alpha = 3.0\,\mathrm{s}$. In the adaptive control case, initialize the RLS algorithm with a **non-zero** estimate $\hat{\boldsymbol{p}}(0)$.

- Prevent the dangerous behavior of the adaptive control system by appropriate modification of the synthesis block – apply the supervision/safety nets with respect to a value of estimate $\hat{k}$.

- Run the MIAC control system and analyze the resultant control quality for both types of a reference trajectory generated by the RSG block – see (17)-(18) – using parameters: $Y_r = 0.15\,\mathrm{rad/s}$, $\omega_r = 0.25\,\mathrm{rad/s}$.
  Important: for the analysis purposes check the time plots of the tracking error e as well as the control input u, and compare output y with trajectory y_r on the same plot; check also time plots of estimates $\hat{\boldsymbol{p}}$ and controller parameters $\boldsymbol{w}$.
  Repeat simulations for

  $$\texttt{sigma2e} \in \{0.0;\ 0.01;\ 0.1\}. \tag{20}$$

  Does the system satisfy performance requirements R2 and R3 in all the cases? Is the control signal u constrained to the physically meaningful range $[-\pi; \pi]$?

- Check how saturation of the control signal to the range $[-\pi; \pi]$ influences the overall control performance (apply the `Saturation` block in series on the output of the controller with $u_{\max} = -u_{\min} = \pi$, see Fig. 2).

- Check how the value of sampling interval T_a (relative to T_c) influences quality of the adaptive control process.

- Check how the initial value of covariance matrix $\boldsymbol{P}(0) = \rho \boldsymbol{I}$, $\rho > 0$, influences quality of the adaptive control process.

□