Laboratory of Adaptive Control

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E6 Active/Adaptive Disturbance Rejection Control (ADRC)

The exercise is devoted to the control design problem in the ADRC (Active/Adaptive Disturbance Rejection Control) scheme for the exemplary plant and to verification of the designed control system in the Matlab-Simulink environment. In the ADRC control scheme, the adaptation process results from the appropriate on-line update of an additional control signal which is responsible for compensation of the so-called total disturbance encompassing all the unknown and uncertain terms of the plant model.

1 Description of the plant

Let us consider the process of rolling motion for the delta-wing aircraft presented in Fig. 1. Thanks to the differential deflection δ_a (expressed in [rad]) of ailerons on the left and right

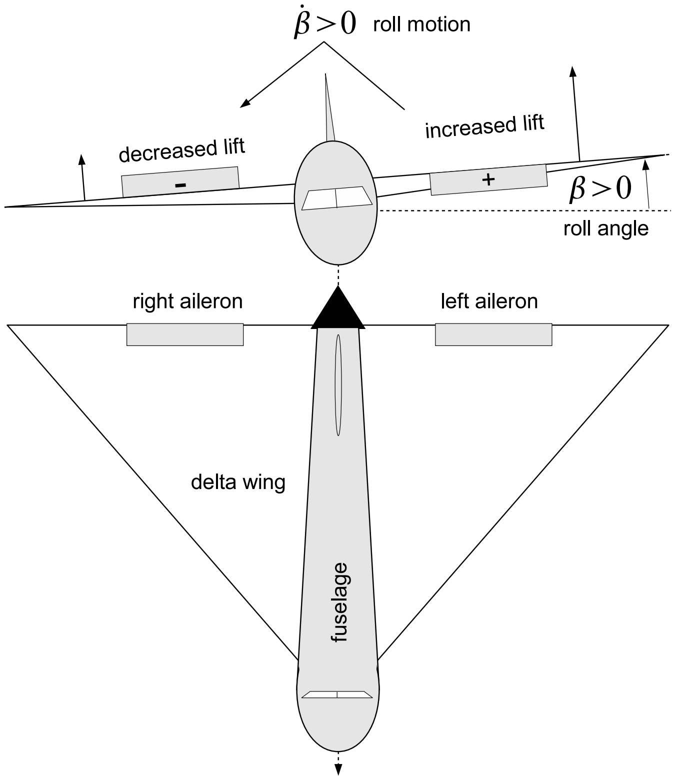

Figure 1: A view of the delta-wing aircraft in the rolling motion with angular velocity $\dot{\beta}$ (based on E. Lavretsky, K. A. Wise: Robust and Adaptive Control with Aerospace Applications, Springer, London, 2013)

sides of the aircraft body, it is possible to change the aircraft roll-angle β expressed in [rad]. One can approximate the roll-angle dynamics by the following nonlinear differential equation

$$\ddot{\beta} = \theta_{10}\beta + \theta_{20}\dot{\beta} + \theta_{60}(\delta_a + d) + (\theta_{30}\,|\beta| + \theta_{40}\,|\dot{\beta}\,|)\dot{\beta} + \theta_{50}\beta^3, \tag{1}$$

where $\theta_{10}, \theta_{20}, \ldots, \theta_{60}$ are the true parameters of the plant, while the term d represents some external disturbance which affects the plant through the input channel. From now on, we assume what follows:

A1. the only measurable signals are the control input $u \triangleq \delta_a$ and the plant output, that is, the roll angle: $y \triangleq \beta$,

A2. the values of parameters $\theta_{10} = -0.018$ and $\theta_{20} = 0.015$ are perfectly known upon the a priori knowledge,

A3. upon the a priori knowledge it is known that the input gain $\theta_{60} \in [0.05, 0.80]$,

A4. values of parameters θ_{30}, θ_{40}, and θ_{50} are unknown,

A5. the external disturbance d is bounded but it is unmeasurable, unknown, and can be time varying.

Note that values of control input $u = \delta_a$ must be inherently constrained to a (subset of) range $[-\pi; \pi]$ rad due to physical interpretation of δ_a.

2 Control performance requirements

We are interested in designing the ADRC control system for the roll-angle dynamics represented by equation (1) which, under assumptions A1 to A5, guarantees satisfaction of the following prescribed performance requirements:

R1. signal $y_r(t) = \beta_r(t)$ is a bounded time-varying reference trajectory for the aircraft roll angle such that $\dot{y}_r(t)$ and $\ddot{y}_r(t)$ exist and are bounded,

R2. tracking error $e(t) \triangleq y_r(t) - y(t)$ converges with no overshoot to an arbitrary small vicinity of zero, that is $|e(t)| \leqslant \epsilon$ for $t \to \infty$, where $\epsilon \geqslant 0$ is a sufficiently small constant,

R3. settling time $T_{s1\%}$ of the closed-loop system satisfies $T_{s1\%} \approx \alpha$ for $\alpha > 0$ expressed in [s].

3 Control system design

3.1 Step 1: description of the plant model in the extended-state space

The roll-angle dynamics (1) represents a highly uncertain system which can be rewritten as

$$\ddot{\beta} = \theta_{10}\beta + \theta_{20}\dot{\beta} + \hat{\theta}_6\delta_a + F(\beta, \dot{\beta}, \delta_a, d), \tag{2}$$

where

$$F(\beta, \dot{\beta}, \delta_a, d) = (\theta_{30}\,|\beta| + \theta_{40}\,|\dot{\beta}\,|)\dot{\beta} + \theta_{50}\beta^3 + \theta_{60}d + (\theta_{60} - \hat{\theta}_6)\delta_a$$

will be called the *total disturbance* since it aggregates all the unknown or uncertain terms of the plant model. In order to design the ADRC control system, we need to write the plant dynamics (2) in the form of an extended-state space model. Let us introduce the *extended state*

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \triangleq \begin{bmatrix} \beta \\ \dot{\beta} \\ F \end{bmatrix} \tag{3}$$

where the third state variable is equal to the total disturbance. The expression *extended state* means that the natural state of the second order dynamics (2) is extended by an additional (artificial) state variable equal to the total disturbance. Justification of this extension will be provided in Sections 3.2 and 3.3. Upon equation (2), and taking $u \triangleq \delta_a$ (see assumption A1), one derives the extended-state space model in the following form

$$\dot{\boldsymbol{x}} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ \theta_{10} & \theta_{20} & 1 \\ 0 & 0 & 0 \end{bmatrix}}_{\boldsymbol{A}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \hat{\theta}_6 \\ 0 \end{bmatrix}}_{\boldsymbol{B}} u + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \dot{F} \tag{4}$$

which shall be complemented by the output equation (according to assumption A1)

$$y = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{\boldsymbol{C}} \boldsymbol{x} = x_1. \tag{5}$$

3.2 Step 2: design of the controller structure

A nominal controller in the ADRC scheme consists of two loops: the total disturbance compensation loop represented by component u_{cn}, and the outer feedback-feedforward loop represented by component u^*. The nominal ADRC control signal is defined as follows

$$u_n \triangleq \frac{u^* + u_{cn}}{\hat{\theta}_6}, \qquad u_{cn} \triangleq -F, \tag{6}$$

where the term u^* will be designed in Section 3.4. The general idea characteristic for the control law (6) relies on a compensation for the unknown term F in dynamics (2), and then on designing the outer-loop control u^* for the compensated plant model. A practical efficiency of the ADRC controller substantially depends on the effectiveness of the total disturbance compensation process. Since the total disturbance F is unknown and unmeasurable, the nominal control law (6) cannot be applied in practice. Therefore, one proposes a practical version of ADRC control law in the form

$$u \triangleq \frac{u^* + u_c}{\hat{\theta}_6}, \qquad u_c \triangleq -\hat{F}, \tag{7}$$

where $\hat{F}$ is an estimate of the total disturbance. Since F is a third variable of the extended state (3), it is possible to design a state observer which will allow estimating F and next utilizing it in the control law (7).

3.3 Step 3: design of the extended-state observer (LESO)

In order to estimate the total disturbance term, one introduces the Linear Extended-State Observer (LESO), which corresponds to the classical Luenberger observer defined for the extended state (3), that is,

$$\begin{aligned} \dot{\hat{\boldsymbol{x}}} &= \boldsymbol{A}\hat{\boldsymbol{x}} + \boldsymbol{B}u + \boldsymbol{L}(y - \boldsymbol{C}\hat{\boldsymbol{x}}) \\ &= \underbrace{(\boldsymbol{A} - \boldsymbol{L}\boldsymbol{C})}_{\boldsymbol{\Gamma}} \hat{\boldsymbol{x}} + \boldsymbol{B}u + \boldsymbol{L}y, \end{aligned} \tag{8}$$

where matrices $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ are taken from (4) and (5), respectively, while

$$\boldsymbol{L} = \begin{bmatrix} l_1 & l_2 & l_3 \end{bmatrix}^\top \tag{9}$$

is the observer gain vector which has to be designed yet. The matrix

$$\boldsymbol{\Gamma} = \boldsymbol{A} - \boldsymbol{L}\boldsymbol{C} = \begin{bmatrix} -l_1 & 1 & 0 \\ \theta_{10} - l_2 & \theta_{20} & 1 \\ -l_3 & 0 & 0 \end{bmatrix} \tag{10}$$

must have all the eigenvalues in the left-half complex plane. Determination of the observer gains is possible by appropriate locating the eigenvalues of matrix $\mathbf{\Gamma}$ using the design condition

$$\det(\lambda \boldsymbol{I} - \mathbf{\Gamma}) = W_o(\lambda), \qquad W_o(\lambda) \triangleq (\lambda + \omega_o)^3 = \lambda^3 + 3\omega_o\lambda^2 + 3\omega_o^2\lambda + \omega_o^3 \tag{11}$$

where $W_o(\lambda)$ is the desired polynomial guaranteeing location of a triple real eigenvalue at $\lambda = -\omega_o$ for some prescribed frequency $\omega_o > 0$. Since

$$\det(\lambda \boldsymbol{I} - \mathbf{\Gamma}) = \lambda^3 + (l_1 - \theta_{20})\lambda^2 + (l_2 - \theta_{10} - l_1\theta_{20})\lambda + l_3, \tag{12}$$

thus by comparing the corresponding coefficients of $W_o(\lambda)$ and polynomial (12) one gets the solution:

$$l_1 = 3\omega_o + \theta_{20}, \tag{13}$$

$$l_2 = 3\omega_o^2 + \theta_{10} + l_1\theta_{20}, \tag{14}$$

$$l_3 = \omega_o^3. \tag{15}$$

Equation (8) determines LESO for the continuous time domain. To obtain a discrete time version of LESO, one can discretize equation (8) using, e.g., the Euler-Forward method which yields

$$\hat{\boldsymbol{x}}(n) = \underbrace{(\boldsymbol{I} + T_a\mathbf{\Gamma})}_{\mathbf{\Gamma}_a}\hat{\boldsymbol{x}}(n-1) + \underbrace{T_a\boldsymbol{B}}_{\boldsymbol{B}_a}u(n-1) + \underbrace{T_a\boldsymbol{L}}_{\boldsymbol{L}_a}y(n-1), \tag{16}$$

where $T_a > 0$ is a sampling time used for the estimation process.

3.4 Step 4: design of the outer-loop control component u^*

Since the total disturbance can be on-line estimated by LESO, one may replace $\hat{F}$ in definition (7) with the estimate $\hat{x}_3$ provided by a solution of (8) or (16). After substituting (7) into (4) one gets the dynamics

$$\dot{x}_2 = \theta_{10}x_1 + \theta_{20}x_2 + x_3 + \hat{\theta}_6\frac{u^* - \hat{x}_3}{\hat{\theta}_6} = \theta_{10}x_1 + \theta_{20}x_2 + u^* + (x_3 - \hat{x}_3) \tag{17}$$

Design of the outer-loop control u^* will be conducted for the case of *perfect compensation (cancellation)* of the total disturbance, that is, for $x_3 - \hat{x}_3 \equiv 0$. In these ideal conditions holds

$$\dot{x}_2 = \theta_{10}x_1 + \theta_{20}x_2 + u^* \quad \Rightarrow \quad \ddot{\beta} = \theta_{10}\beta + \theta_{20}\dot{\beta} + u^*. \tag{18}$$

By definition of the tracking error we can write

$$e = \beta_r - \beta \quad \Rightarrow \quad \beta = \beta_r - e \tag{19}$$

$$\dot{e} = \dot{\beta}_r - \dot{\beta} \quad \Rightarrow \quad \dot{\beta} = \dot{\beta}_r - \dot{e} \tag{20}$$

$$\ddot{e} = \ddot{\beta}_r - \ddot{\beta} \quad \Rightarrow \quad \ddot{\beta} = \ddot{\beta}_r - \ddot{e} \tag{21}$$

which allow rewriting the compensated dynamics (18) in the form

$$\ddot{\beta}_r - \ddot{e} = \theta_{10}(\beta_r - e) + \theta_{20}(\dot{\beta}_r - \dot{e}) + u^* \tag{22}$$

or, after simple reordering the particular terms, as

$$\ddot{e} - \theta_{20}\dot{e} - \theta_{10}e = \ddot{\beta}_r - \theta_{20}\dot{\beta}_r - \theta_{10}\beta_r - u^*. \tag{23}$$

Let us define the outer-loop control in a way which ensures that the closed-loop error dynamics is bounded and asymptotically convergent to zero. To this aim, we propose a definition

$$u^* \triangleq \underbrace{\ddot{\beta}_r - \theta_{20}\dot{\beta}_r - \theta_{10}\beta_r}_{\text{feedforward}} + \underbrace{k_d\dot{e} + k_p e}_{\text{PD feedback}} \tag{24}$$

which consists of the feedforward term (depending on the reference signals) and the PD feedback term with two controller gains $k_p, k_d > 0$ which have to be designed yet. By substituting the control law (24) into (23) yields the closed-loop error dynamics

$$\ddot{e} + (k_d - \theta_{20})\dot{e} + (k_p - \theta_{10})e = 0 \tag{25}$$

which is asymptotically stable if the two coefficients near the terms e and $\dot{e}$ are positive. In order to satisfy the performance requirements R2 and R3, one proposes to select the controller gains k_p and k_d according to the following design condition

$$s^2 + (k_d - \theta_{20})s + (k_p - \theta_{10}) = W_c(s), \qquad W_c(s) \triangleq (s + \omega_c)^2 = s^2 + 2\omega_c s + \omega_c^2 \tag{26}$$

where $W_c(s)$ is the desired polynomial guaranteeing location of a double real pole at $s = -\omega_c$ for some prescribed frequency $\omega_c > 0$. By comparing the corresponding coefficients of the polynomials in (26) one gets the synthesis equations

$$k_p = \omega_c^2 + \theta_{10}, \tag{27}$$

$$k_d = 2\omega_c + \theta_{20}. \tag{28}$$

Note that frequency ω_c should be appropriately related to the frequency ω_o used in the LESO design (see (11)). A general rule is

$$\omega_c \ll \omega_o \tag{29}$$

to assure that the convergence of estimation error $\hat{\boldsymbol{x}}(t) - \boldsymbol{x}(t)$ will be substantially faster than the convergence of tracking error $e(t)$. Additionally, upon requirement R3 one shall select

$$\omega_c \approx \frac{2\pi}{\alpha} \quad \text{because} \quad T_{s1\%} \approx \frac{2\pi}{\omega_c}. \tag{30}$$

Combination of rule (29) and condition (30) gives the proposed synthesis equations

$$\omega_c \approx \frac{2\pi}{\alpha}, \qquad \omega_o = \frac{\omega_c}{\mu}, \qquad 0 < \mu \ll 1 \tag{31}$$

where μ is a sufficiently small scaling factor. Worth to emphasize here that the synthesis equations (27)-(28) and tuning rules (31) guarantee satisfaction of requirement R3 with no-overshooting response only in the perfect compensation conditions. Thus, if the compensation of the total disturbance is not perfect (a practical case), the requirement R3 will be met only approximately.

Summarizing, the ADRC control law is a combination of definition (7) with (24), and takes the final form

$$u = \left[\ddot{\beta}_r - \theta_{20}\dot{\beta}_r - \theta_{10}\beta_r + k_d(\dot{\beta}_r - \hat{x}_2) + k_p(\beta_r - \beta) - \hat{x}_3\right] / \hat{\theta}_6, \tag{32}$$

where $\hat{x}_2$ and $\hat{x}_3$ are taken from LESO. The term $\ddot{\beta}_r - \theta_{20}\dot{\beta}_r - \theta_{10}\beta_r$ is responsible for the feedforward control, the term $k_d(\dot{\beta}_r - \hat{x}_2) + k_p(\beta_r - \beta)$ is the PD feedback control, while $\hat{x}_3$ is responsible for a compensation of the total disturbance. According to assumption A1, the D part of the PD control component has been rewritten by using the estimate $\hat{x}_2$, because signal $\dot{\beta}$ is not measurable and it cannot be used in the controller. Moreover, in practical applications it is recommended to post-process the control signal (32) with the saturation function

$$\text{Sat}(u, u_m) \triangleq \min\{|u|, u_m\} \cdot \text{sign}(u), \tag{33}$$

where $u_m > 0$ is a prescribed saturation level of the control signal (in the case of system (1), one shall take $u_m \leqslant \pi$). A resultant scheme of the ADRC control system corresponding to the control law (32) followed by the saturation block is presented in Figure 2.

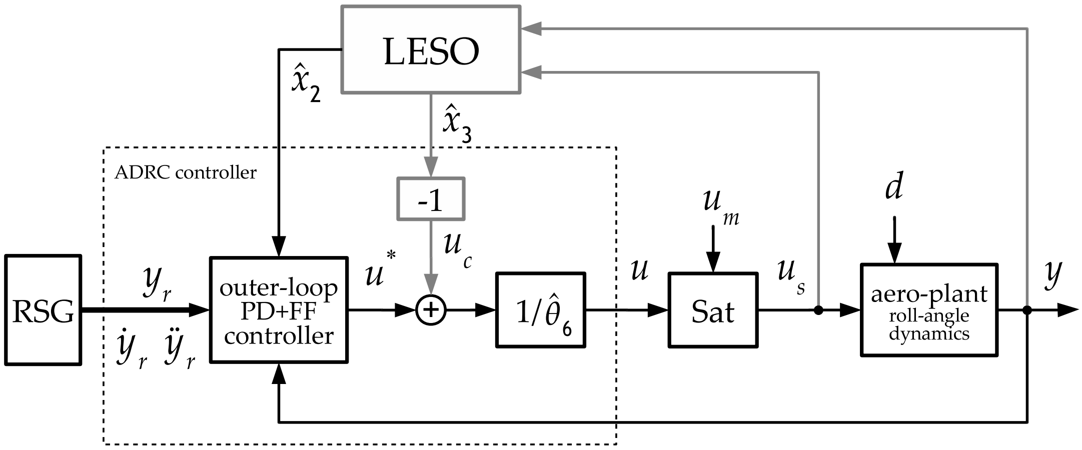

Figure 2: Block scheme of the ADRC system for the aero-plant; the blocks and arrows highlighted in gray constitute the adaptive loop in the control system while the black ones correspond to the conventional part of the control system (RSG = Reference Signals Generator)

3.1 Reference trajectory tracking in the ADRC system.

- Open the file `DeltaWingPlantADRC.mdl` which contains the plant described by dynamics (1) and the reference signal generator (RSG). The RSG block produces two types of the reference trajectory $y_r(t)$ and its time derivatives $\dot{y}_r(t)$, $\ddot{y}_r(t)$:

$$\text{TYPE 1:} \quad y_r(t) \triangleq Y_r \sin(\omega_r t), \tag{34}$$

$$\text{TYPE 2:} \quad y_r(t) \triangleq H(s)[Y_r(t)\text{rect}(\omega_r t)], \tag{35}$$

  where $\text{rect}(\omega_r t)$ represents a rectangular signal with unit amplitude and frequency ω_r rad/s, $H(s)$ is a prescribed third order linear filter, while $Y_r(t)$ is a time-varying amplitude.

- On the scheme in file `DeltaWingPlantADRC.mdl` implement the estimator LESO in the continuous-time-domain form determined by (8), the outer-loop PD+FF controller (24), and close the loop according to the control law (32) followed by the saturation block. Select the following parameters: $u_m = \pi$, $\alpha = 2.0\,\text{s}$, $\hat{\theta}_6$ from a middle point of the known range $[0.05, 0.8]$, and $\mu = 0.2$.

- Run the ADRC system for the external disturbance $d(t) \equiv 0$ and the plant initial conditions $\beta(0) = 0.4\,\text{rad}$, $\dot{\beta}(0) = 0$ (to the latter aim, one needs to appropriately initialize two global variables `beta0` and `betap0`). Analyze the resultant control quality for both types of a reference trajectory generated by the RSG block – see (34)-(35) – using the default parameters (read them out from inside of the RSG blocks).
  Important: for the analysis purposes check the time plots of the tracking error $e(t)$ as well as the control signal $u(t)$, and compare state $\boldsymbol{x}(t)$ with its estimate $\hat{\boldsymbol{x}}(t)$ computed by LESO; compare the reference signal $y_r(t)$ with the plant output on the same plot; check also behavior of the estimate $\hat{F}(t)$.
  Repeat and analyze simulations for

$$\mu \in \{0.5;\ 0.1;\ 0.03\}. \tag{36}$$

  Does the system satisfy performance requirements R2 and R3 in all the cases? How does the decreasing value of μ influence the control performance and the control signal? What effects do you expect after adding a stochastic noise to signal y?

- Compare the control performance of the full ADRC controller with the performance obtained when the compensation (adaptation) loop is open (by forcing $\hat{x}_3(t) \equiv 0$).

- Turn on the external disturbance $d(t)$, which can be selected inside a block of the plant. Check the control performance of the ADRC system in these conditions. Repeat the simulations for μ taken from the set determined by (36). Compare the control performance when the compensation (adaptation) loop is open.

- Replace the continuous-time LESO with its discrete-time counterpart represented by (16). Check the influence of the sampling time T_a used for computations of the discrete-time LESO on the overall tracking performance for the following set of sampling time values:

$$T_a \in \{0.04;\ 0.03;\ 0.01\}\, s. \tag{37}$$

  Note: the above simulations conduct using $\mu = 0.1$.

□