Topics for the exam: analysis

You should be able to explain, in writing, the following.

1. The principle of induction.
2. Concept of a real number as nonterminating decimal.
3. Function: definition and graph.
4. Inverse function.
5. Inverse trig functions.
6. Composite function.
7. Limit of a function.
8. Continuity of a function.
9. Derivative of a function.
10. Secant line and tangent line.
11. Product Rule, Quotient Rule.
12. Chain Rule.
13. Derivative of the inverse function.
14. Rolle's theorem and the mean value theorem.
15. Local and global extremum of a function.
16. Concavity of a function.
17. Implicit differentiation and related rates.
18. Indefinite forms. L'Hospital's Rule.
19. Antiderivative and indefinite integral of a function.
20. Integration by substitution and by parts.
21. Rational function and its decomposition to polynomial and partial fractions.
22. Definite integrals and Riemann sums.
23. Fundamental Theorem of Calculus.
24. Applications of definite integral to geometry (area, volume, and arc length).
25. Infinite number series.
26. Tests for the convergence/divergence of number series.
27. Power series and the radius and region of its convergence.
28. Taylor/Maclaurin series expansion of a function near a point.

You should also be competent in the following skills:

1. Use induction to prove equalities and inequalities.
2. Find the formula and graph of the inverse function of a given function, if possible.
3. Find limits using basic rules, squeeze theorem, and limits of $\frac{\sin x}{x}$, $\left(1+\frac{1}{x}\right)^x$
4. Use both the limit definition and rules of differentiation to differentiate functions.
5. Sketch the graph of a function using asymptotes, critical points, the derivative test for increasing/decreasing functions, and concavity.
6. Apply differentiation to solve applied max/min problems.
7. Apply differentiation to solve related rates problems.
8. Evaluate certain indefinite forms by using L'Hospital's Rule.
9. Evaluate indefinite integrals by using basic rules, substitution, integration by parts, reduction formulas and decomposition to partial fractions.
10. Evaluate definite integrals both by using Riemann sums and by using the Fundamental Theorem of Calculus.
11. Apply integration to compute arc lengths, volumes of revolution and surface areas of revolution.
12. Determine convergence/divergence of improper integrals and evaluate convergent improper integrals.
13. Determine the convergence/divergence of an infinite series and find the Taylor series expansion of a function near a point.