

Below are the problems from the last year's Analysis exam

You can expect something similar.

1.(8 pts) Formulate the Principle of Induction. Prove by induction that for any $a, q \in \mathbb{R}$, $q \neq 1$ and each natural n there is the equality:

$$a + aq + aq^2 + \cdots + aq^{n-1} = a \frac{q^n - 1}{q - 1}.$$

2.(6 pts) Write the definition of the derivative of $f(x)$ at a point x_0 . Explain it geometrically. Using the definition of the derivative find the derivative of $f(x) = 2x^2 - x$ at $x = 1$.

3.(8 pts) a) Write the First Derivative Test for extrema.

b) Find the dimensions of the rectangle with largest area that can be inscribed in a semicircle of radius R , if two vertices lie on the diameter. Give comments to the calculations you do.

4.(8 pts) Explain the method of partial fraction decomposition and apply it to

$$f(x) = \frac{3x^2 + 2x - 2}{x^3 - 1}.$$

5.(8 pts) Explain integration by substitution and integration by part. Evaluate $\int x \ln x \, dx$ and $\int (\cos^2 x - 2 \cos x) \sin x \, dx$.

6.(6 pts) Draw the region bounded by the curves: $y = x^2$, $y = 2x^2$, $y = 8$ and find its area.

7.(6 pts) What does it mean that a series $\sum_{n=1}^{\infty} a_n$ is convergent? (Give the definition). Determine whether $\sum_{n=1}^{\infty} \frac{n^2 + n}{3^n(n^2 + 3n)}$ is convergent and write full text of the convergence test that you have used.