Homework 4: systems with a parameter, Cramer's Rule, vector space

1. Express the system in the form $\mathbf{A}\mathbf{x} = \mathbf{b}$. Solve it by first finding $\mathbf{A}^{-1}$.

$$\begin{array}{rcrcrcr} x & + & 2y & + & 3z & = & 3 \\ x & + & y & + & z & = & 1 \\ 3x & - & y & + & 2z & = & 1 \end{array}$$

2. Solve by Cramer's Rule

$$\left\{ \begin{array}{rcrcrcr} x_1 & + & x_2 & + & x_3 & = & 1 \\ x_1 & + & 2x_2 & + & 3x_3 & = & 2 \\ x_1 & + & 4x_2 & + & 10x_3 & = & -1 \end{array} \right.$$

3. For which value of a the following system has a solution? Find the solution.

$$\left\{ \begin{array}{rcrcrcr} x & + & ay & + & 2z & = & 4 \\ ax & + & y & & & = & 1 \\ ax & + & y & - & 2z & = & 1 \end{array} \right.$$

4. Determine whether $(4, 6, 6)$ is a linear combination of the vectors $\mathbf{v}_1 = (1, 2, -1)$ and $\mathbf{v}_2 = (3, 5, 2)$ in $\mathbb{R}^3$.
5. Prove that $\{(1, 1), (-1, 1)\}$ is a basis for $\mathbb{R}^2$.
Hint. You must show that: 1. vectors are linearly independent; 2. any vector $(x, y) \in \mathbb{R}^2$ is a linear combination of them.

Please write the solutions clearly (by hand) on A4 paper and give it to me before 15/01/2019. Every solution will be given 1 point (correct, minor error possible), 0.5 pt. (good idea, but not all correct), 0 pt. (nothing worthy). The maximum for this homework is 5 pts.