Derivatives: you must know this!

Rules for Derivatives

$$(c)' = 0 \tag{1}$$
$$(x^\alpha)' = \alpha x^{\alpha-1}, \quad \text{where } \alpha \in \mathbb{R} \tag{2}$$
$$(\sin x)' = \cos x \tag{3}$$
$$(\cos x)' = -\sin x \tag{4}$$
$$(\tan x)' = \frac{1}{\cos^2 x} = 1 + \tan^2 x \tag{5}$$
$$(\cot x)' = \frac{-1}{\sin^2 x} = -1 - \cot^2 x \tag{6}$$
$$(a^x)' = a^x \ln a \quad \text{where } 0 < a \neq 1 \tag{7}$$
$$(e^x)' = e^x \tag{8}$$
$$(\log_a x)' = \frac{1}{x \ln a} \tag{9}$$
$$(\ln x)' = \frac{1}{x} \tag{10}$$
$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \tag{11}$$
$$(\arccos x)' = \frac{-1}{\sqrt{1-x^2}} \tag{12}$$
$$(\arctan x)' = \frac{1}{1+x^2} \tag{13}$$
$$(\operatorname{arccot} x)' = \frac{-1}{1+x^2} \tag{14}$$
$$(\sinh x)' = \cosh x \tag{15}$$
$$(\cosh x)' = \sinh x \tag{16}$$
$$(\tanh x)' = \frac{1}{\cosh^2 x} \tag{17}$$
$$(\coth x)' = \frac{-1}{\sinh^2 x} \tag{18}$$

"Arithmetic" Rules:

1. $(f+g)'(x) = f'(x) + g'(x)$;
2. $(f-g)'(x) = f'(x) - g'(x)$;
3. $(cf)'(x) = cf'(x)$, where $c \in \mathbb{R}$;
4. $(fg)'(x) = f'(x)g(x) + f(x)g'(x)$;
5. $\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x)-f(x)g'(x)}{g^2(x)}$, provided $g(x) \neq 0$.

The Chain Rule:

$$(g \circ f)'(x) = g'\left(f(x)\right) f'(x).$$

Derivative as a Slope of the Tangent Line

If $y = f(x)$ is differentiable at x_0, then $f'(x_0)$ is a slope of the tangent line at $(x_0, f(x_0))$. The equation of the tangent line is:

$$y = f(x_0) + f'(x_0)(x - x_0).$$

Higher Derivatives
$y^{(n)} = (y^{(n-1)})'$ dla $n = 2, 3, 4, \ldots$.
By agreement: $y^{(0)} = y$.
For small orders we write: y'', y''', y^{IV}, y^{V}, etc.

Implicit Differentiation
If $y = f(x)$ is implicitly defined by some equality, then we don't need to solve for y. We differentiate the whole equality using Chain Rule for terms with y. For example if

$$y^3 - 3x^2y^2 - 5x = x\sin y$$

then

$$3y^2y' - (6xy^2 + 6x^2yy') - 5 = \sin y + x\cos y \cdot y',$$

and now solve for y':

$$y' = \frac{6xy^2 + 5 + \sin y}{3y^2 - 6xy^2 - x\cos y}.$$